

"THE SAMPLE VARIANCE F-TEST WORKSHOP"

Introduction

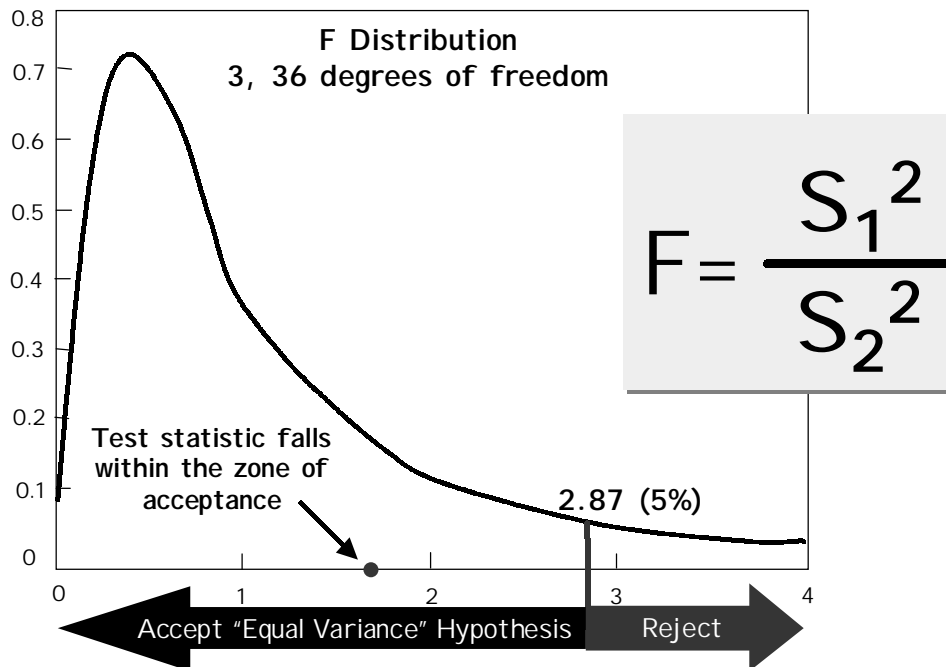
We are now very familiar with the concept of **Hypothesis Testing**, and have investigated in detail the application of the t -test to determine if the differences observed in the mean of independent samples can be considered statistically significant.

You will recall, however, that the capability of a process can be characterized by two things, and not merely just the mean. We can measure where we hit the target relative to a goal, but we also have to understand how consistent we are in hitting the same location on the target. Remember: Accuracy and Precision are vital for six sigma levels of performance.

We need tools that enable us to compare variance, just as we did for the mean or average with the t -test. The **F-test** provides a tool for comparing variances, as we will discover in this workshop.

Workshop Objectives

After completing this workshop, you will understand the principles behind the F-test, the importance of knowing if **variance between samples is statistically significant**, and how to apply the F-test when analyzing those samples.



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Explanation

The difference between two variances can be studied using another sampling called the F distribution. Just as with the t -test, you calculate a value for F using information from your samples, and compare this **F statistic** to the sampling distribution (table) to determine if the value falls within the zone of acceptance, or in the critical region for rejection of the hypothesis that the variances are equal.

The F distribution is **very closely related to the chi-square distribution**, and provides us with important information when we compare the variance of two independent random samples from a normally distributed population.

To perform your F-test, all you require is knowledge of the **variance** of both samples, as well as the **sample size**, as you will discover in subsequent pages of this workshop.

Equality of Variance and ANOVA

You will remember in our previous "Fishy Story" during the Sample Means T-Test workshop, there was a section on **Equality of Variance**. While not practically significant in the t -test, we had the choice of using **pooled standard deviation** in our calculations if we were certain the variances were equal.

The F-Test allows us to make that determination, and has important implications in **Analysis of Variance (ANOVA) and Regression**, which will be the subject of future workshops.

The F Distribution

As depicted in the illustration on the preceding page, the F distribution is **related to the chi-square distribution** and has important applications in statistics. If X and Y are independent chi-square random variables with **degrees of freedom m and n** , then the random variable is said to have the F Distribution with m and n degrees of freedom.

$$F = \frac{\chi_1^2 / v_1}{\chi_2^2 / v_2}$$



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- ↪ In order to describe a given F Distribution, you must be able to specify the degrees of freedom for the numerator and the degrees of freedom for the denominator, ($DF = n - 1$, where n is your sample size).
- ☞ The chi-square, t , and F Distributions are all related to the normal distribution and are used extensively in statistical inference.

One-Tail versus Two-Tail F-Tests

Our hypothesis can test for one of two things: That the variances of two samples are equal, ($H_0: \sigma_1 = \sigma_2$, $H_a: \sigma_1 \neq \sigma_2$), or that the variance of one sample is greater than/less than the other, ($H_0: \sigma_1 = \sigma_2$, $H_a: \sigma_1 < \sigma_2$).

The former requires a **two-tailed test** since you are not **stating which one will be larger**. The latter is **testing which variance is larger**, and in that instance, the **one-tailed test** is appropriate.

- ↪ When a **two-tailed test** is required, you must **double the probabilities** when you use the F table., (ex. 5% becomes 10%, 1% becomes 2%).

"Fishy Story - Part II"

Let's recall our example from the Sample Means T-Test workshop. Our fishers were on a quest to purchase cottage property in Northern Canada. At one point, they had two samples of fifteen fish that they had caught from the lake.

Their first catch measured 7.5, 8.2, 8.1, 8.4, 7.1, 7.3, 7.1, 7.8, 8.0, 7.3, 7.3, 7.9, 7.8, 8.1, and 7.6 inches in length respectively. The next month, their catch is 7.9, 8.4, 8.0, 7.8, 7.5, 7.2, 7.4, 7.3, 8.1, 7.6, 7.7, 8.1, 7.7, 6.8, and 7.5 inches. Using the t -test, we were able to determine that any observed difference in average fish length (mean) was not statistically significant. In our test, we had **assumed unequal variances**.

- ☞ Our hypothesis is stated as $H_0: \sigma_1 = \sigma_2$, $H_a: \sigma_1 \neq \sigma_2$, which means we are applying the two-tailed test.

While for practical purposes in the t -test, we are fine using the unequal variances assumption, could we not apply the F-Test to determine if the variances were, in fact, equal?

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Calculate the F ratio

Our first step is to determine the variance for our samples and then calculate the F statistic by **dividing the higher value variance by the lower value**.

In the case of our first catch, the variance is determined to be .1898, and for the second catch of fish, .1757. Therefore, $F = 1.08$

⇒ An F ratio close to 1.0 indicates that the two samples have similar variances.

$$\begin{aligned} F &= \frac{S_1^2}{S_2^2} \\ &= \frac{.1898}{.1757} \\ &= 1.08 \end{aligned}$$

Let's now compare this statistic to the critical points of the F distribution table, using the degrees of freedom associated with each sample.

We know that the degrees of freedom for each sample can be determined as the sample size, n , minus one, $(n-1)$. Therefore, the degrees of freedom for each sample must be $15-1 = 14$.

By consulting the "Table of Critical Points for the F Distribution - 5% Significance Level", reference the DF for the numerator, and the DF for the denominator.

$$F = \frac{S_1^2}{S_2^2}$$

The diagram shows the F-ratio formula with arrows pointing to the components: 'numerator' points to S_1^2 , 'larger value' points to S_1^2 , 'denominator' points to S_2^2 , and 'smaller value' points to S_2^2 .

Our first catch, which had a larger value for variance, (.1898), assumes the role of the numerator, while the second catch with a variance of .1757 is

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positioned as the denominator. Our numerator, therefore, has a DF of 14, as does our denominator.

DF denom.	Degrees of Freedom for Numerator																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	25	30								
1	161	199	216	225	230	234	237	239	241	242	243	244	245	245	246	246	247	247	248	248	249	250								
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5								
3	10.1	9.6	9.3	9.1	9.0	8.9	8.9	8.8	8.8	8.8	8.8	8.7	8.7	8.7	8.7	8.7	8.7	8.7	8.7	8.7	8.6	8.6								
4	7.7	6.9	6.6	6.4	6.3	6.2	6.1	6.0	6.0	6.0	5.9	5.9	5.9	5.9	5.9	5.8	5.8	5.8	5.8	5.8	5.8	5.7								
5	6.6	5.8	5.4	5.2	5.1	5.0	4.9	4.8	4.8	4.7	4.7	4.7	4.7	4.6	4.6	4.6	4.6	4.6	4.6	4.6	4.5	4.5								
6	6.0	5.1	4.8	4.5	4.4	4.3	4.2	4.1	4.1	4.1	4.0	4.0	4.0	4.0	3.9	3.9	3.9	3.9	3.9	3.9	3.8	3.8								
7	5.6	4.7	4.3	4.1	4.0	3.9	3.8	3.7	3.7	3.6	3.6	3.6	3.6	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.4	3.4								
8	5.3	4.5	4.1	3.8	3.7	3.6	3.5	3.4	3.4	3.3	3.3	3.3	3.3	3.2	3.2	3.2	3.2	3.2	3.2	3.2	3.1	3.1								
9	5.1	4.3	3.9	3.6	3.5	3.4	3.3	3.2	3.2	3.1	3.1	3.1	3.0	3.0	3.0	3.0	3.0	3.0	2.9	2.9	2.9	2.9								
10	5.0	4.1	3.7	3.5	3.3	3.2	3.1	3.1	3.0	3.0	2.9	2.9	2.9	2.9	2.8	2.8	2.8	2.8	2.8	2.8	2.7	2.7								
11	4.8	4.0	3.6	3.4	3.2	3.1	3.0	2.9	2.9	2.9	2.8	2.8	2.8	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.6	2.6								
12	4.7	3.9	3.5	3.3	3.1	3.0	2.9	2.8	2.8	2.8	2.7	2.7	2.7	2.6	2.6	2.6	2.6	2.6	2.6	2.5	2.5	2.5								
13	4.7	3.8	3.4	3.2	3.0	2.9	2.8	2.8	2.7	2.7	2.6	2.6	2.6	2.6	2.5	2.5	2.5	2.5	2.5	2.5	2.4	2.4								
14	4.6	3.7	3.3	3.1	3.0	2.8	2.8	2.7	2.6	2.6	2.6	2.5	2.5	2.5	2.5	2.4	2.4	2.4	2.4	2.4	2.3	2.3								
15	4.5	3.7	3.3	3.1	2.9	2.8	2.7	2.6	2.6	2.5	2.5	2.5	2.4	2.4	2.4	2.4	2.4	2.4	2.3	2.3	2.3	2.2								
20	4.4	3.5	3.1	2.9	2.7	2.6	2.5	2.4	2.4	2.3	2.3	2.3	2.2	2.2	2.2	2.2	2.2	2.2	2.2	2.1	2.1	2.0								
25	4.2	3.4	3.0	2.8	2.6	2.5	2.4	2.3	2.3	2.2	2.2	2.2	2.1	2.1	2.1	2.1	2.1	2.1	2.0	2.0	2.0	1.9								
30	4.2	3.3	2.9	2.7	2.5	2.4	2.3	2.3	2.2	2.2	2.1	2.1	2.1	2.0	2.0	2.0	2.0	2.0	1.9	1.9	1.9	1.8								

While we were already comfortable that the variances of the two samples were equal from a statistical significance point of view since the ratio was so very close to 1.0, we can also see on our table that the critical region for rejection starts at a value of 2.5.

We are well within the acceptance range for accepting the null hypothesis that the variances are equal – that is, at the 10% significance level (5% x 2 = 10% for two-tailed test).

☞ If we were testing if one of the variances was greater than the other, it would be a one-tail test, resulting in a significance level of 5%.

Using MS Excel®

MS Excel has many functions that support statistics. Without having to consult tables, we can use MS Excel to determine critical values for us.

In the previous case, we had a numerator as well as denominator with degrees of freedom equal to 14.

$$\begin{aligned}
 &=FINV(\text{probability}, df_1, df_2) \\
 &=FINV((1 - .95), 14, 14) \\
 &=2.48
 \end{aligned}$$



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We wanted to establish a 95% confidence level for the critical value of F , (F_{crit}). Using the **FINV** function, we return a value of 2.48, (our table on the previous page rounded this value to 2.5).

```
=FDIST(X,df1, df2)  
=FDIST(2.48,14,14)  
=.05
```

Another useful tool within MS Excel is the **FDIST** function. If $FINV(p,...) = X$, then $FDIST(X,...) = p$. To test this, we can use the value 2.48 that was returned by the **FINV** function. It should be no surprise

that MS Excel correctly returns the value of .05, which indicates that there is only a 5% chance that an F random variable will be greater than of 2.48. If we apply this to our “Fishy” example, 1.08 returns a value of .44, a much higher level of probability, and well within the zone of acceptance for the null hypothesis.

Summary

This workshop has provided us with another important sample distribution, which we can use when analyzing the data we have gathered. We understand that the F distribution allows us to make statements about the variances that we observe between two groups of sample data.

We know that the F distribution, like the chi-square and t distribution, is related to normal distribution, and all are used extensively in statistical inference.

By computing the F ratio using the sample variance associated with our two groups of samples, and by knowing the degrees of freedom for each, we know how to reference the F table or use MS Excel® to determine if our statistic falls within the zone of acceptance for our hypothesis that the variances are equal.

Finally, we recognize that this workshop is an introduction to the F distribution, and that it will play a much larger role in future workshops that will introduce Analysis of Variance (ANOVA) and regression.

Champion's Questions

1. Our project team had indicated to me that there was some measured some reduction in the observed variability that was present in the process. You are telling me that the F -test you performed resulted in a



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significance level of .23, thus the observed differences were not statistically significant. Why can't we say we've begun to reduce the variability if that is what we're seeing?!

Quick Quiz (check the appropriate box)

1. The F distribution allows us to study the difference between two _____.
 means random variables
 variances degrees of freedom
 none of the above
2. The F distribution is related to the _____ distribution.
 chi-square t
 normal all of the above
 none of the above
3. In order to describe a given F distribution, you must be able to specify the _____ for both the numerator and the denominator.
 random variables degrees of freedom
 variability probabilities
 none of the above
4. After determining the variance for both samples, you can calculate the F ratio by dividing the _____ value variance by the _____ value.
 higher, lower lower, higher
5. Using the FDI ST where the sample size for the sample with the largest variance is 30, and the sample size for the other sample group is 35, what is the significance level $X = 1.9$?
 .03 .97
 .96 .04
 none of the above

Workshop Exercise

Background Exercise¹: Two growers offer you their crops. Grower A asks a slightly higher price than Grower B, but says that his grapefruits are more uniform in size. To check this assertion, you ask for a random sample of each crop. Each grower sends you a crate of 25 grapefruit. You measure the grapefruit in each sample and obtain the following information.



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- a) The size of the fruit is approximately normally distributed for both samples.
- b) For Grower A, the mean diameter of the fruit is 4.5 inches with a standard deviation of 0.5 inches.
- c) For Grower B, the mean diameter of the fruit is 4.5 inches with a standard deviation of 1 inch.

Workshop Exercise

In-Class Assignment:

1. State an appropriate null hypothesis and alternative for a statistical test.
2. For a significance level of 5%, what is the critical region? (Use the FINV function in MS Excel®, and then check this against the F Distribution Table.
3. Compute the F ratio or statistic and compare this to the critical point identified in the F distribution.
4. Are the results significant? What kind of statement would you make with regard to your original hypothesis?

¹Source: "Statistics, Third Edition", Donald Koosis, Wiley press ISBN 0 471-82720-7

